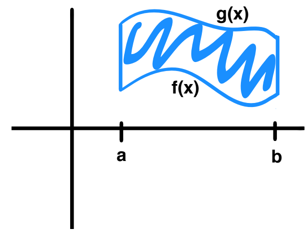
MA 114 MathExcel Worksheet J

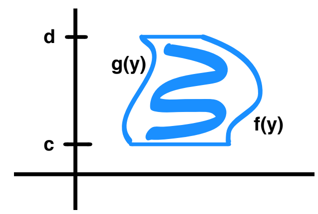
1 Center of Mass

1. Conceptual Understanding:

   (a) Write down the formulas for the coordinates of the centroid of a plate with constant density bounded between $x = a$, $x = b$, $f(x)$, and $g(x)$ as in the figure to the right.



   (b) Write down the formulas for the coordinates of the centroid of a plate with constant density bounded between $y = c$, $y = d$, $f(y)$, and $g(y)$ as in the figure to the right.



2. Find the centroid of the region between $f(x) = x^3$ and $g(x) = \sqrt{x}$.

3. Find the moments and center of mass of the lamina of uniform density ρ occupying the region under $y = x^2$ for $0 \leq x \leq 3$.

4. Find the center of mass for the system of particles of masses 6, 1, 11, and 1 located at the coordinates $(10, 2)$, $(-3, 2)$, $(2, -11)$, and $(4, 4)$, respectively.

5. Find the center of mass of the region created by an isosceles triangle with vertices $(2, 3), (-2, 3), (0, 5)$ on top of the rectangle created by the points $(2, 0), (-2, 0), (2, 3), (-2, 3)$.

2 Parametric Curves

6. Consider the parametric curve: $c(t) = (\cos(2t), \sin^2(t))$ for $0 \leq t \leq 2\pi$. Find the (x, y) coordinates at times $t = 0, \frac{\pi}{4}, \pi$.

7. Find a Cartesian equation for each of the following parametric curves. It may be useful to eliminate the parameter.

   (a) $x = t^2$, $y = t^3 + 1$, $t \in \mathbb{R}$

   (b) $x = \ln(t)$, $y = 2 - t$, $t \geq 0$

   (c) $x = \cos(t)$, $y = \tan(t)$, $0 \leq t \leq 2\pi$. Try to express your answer without trigonometric functions.

8. Graph the following parametric curves; draw an arrow on each curve to specify the direction corresponding to the motion.

   (a) $x = 2t$, $y = t^2, -\infty < t < \infty$

   (b) $x = \frac{t}{\pi}$, $y = \sin(t)$ for $0 \leq t \leq 2\pi$

9. Find parametrizations of the following curves satisfying the given conditions.

   (a) $y = x^2$, $c(0) = (3, 9)$

   (b) $x^2 + y^2 = 4$, $c(0) = (1, \sqrt{3})$

10. Recall that the derivative of a parametric curve $c(t) = (x(t), y(t))$ is given by

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}.$$

For each of the following curves, find $\frac{dy}{dx}$ in two ways. First use the formula above. Then check your work by finding $y = f(x)$ and then differentiating it.

   (a) $c(t) = (2t + 1, 1 - 9t), \quad t \in \mathbb{R}$.

   (b) $c(s) = \left(\frac{s}{2}, \frac{s^2}{4} - s\right), \quad s \in \mathbb{R}$.

   (c) $x = \cos(\theta)$, $y = \cos(\theta) + \sin^2(\theta), \quad 0 \leq \theta \leq 2\pi$.

11. For the following parametric curves, find an equation for the tangent line to the curve at the specified value of the parameter.

   (a) $x = \ln(t), \ y = 1 + t^2$ at $t = 1$.

   (b) $x = \sec(t), \ y = \cot^2(t) - \cos(t)$ at $t = \frac{\pi}{4}$

12. Recall that the second derivative of a parametric function $c(t) = (x(t), y(t))$ is given by

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}.$$

Find $\dfrac{d^2y}{dx^2}$ for the curve $x = t + \sin(t)$, $y = t - \cos(t)$, $t \in (-\infty, \infty)$.